

Attempt 1

```
(define (2nd-max lon)
  (second (sort lon >)))
```

Short, sweet, and $O(n \log n)$

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Attempt 2

- 1 Find the largest value in the list.
- 2 Remove that item from the list.
- 3 Find the largest value in what's left.

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```
(define (2nd-max lon)
  (apply max                ; step 3
    (remove                  ; step 2
      (apply max lon)       ; step 1
      lon)))
```

A little longer, and only $O(n)$.
But it makes three passes...

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Attempt 3

Our argument contains at least two numbers:

(<number> <number> . <list-of-numbers>)

... with the usual definition for a list:

<list-of-numbers>

::= ()

| (<number> . <list-of-numbers>)

.

If we could write a loop, we might...

- Create two local variables,
largest and **2nd-largest**.
- Initialize the variables
using the first two items in the list.
- **Then** look at each item in the rest
of the list to see if it is greater
than either of the two variables
and, if so, update the variables.

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Create an interface procedure to initialize the variables and start the processing:

```
(define (2nd-max lon)
  (2nd-max-tr
   (max (first lon) (second lon))
   (min (first lon) (second lon))
   (rest (rest lon)))))
```

Now we have to write 2nd-max-tr.

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(6 1 2 -3 9 4 -1 2 8 1 2 4)

largest 6

2nd-largest 1

rest (**2** . (-3 9 4 -1 2 8 1 2 4))

.

(6 1 2 -3 9 4 -1 2 8 1 2 4)

largest 6

2nd-largest 2

rest (-3 . (9 4 -1 2 8 1 2 4))

.

(6 1 2 -3 9 4 -1 2 8 1 2 4)

largest 6

2nd-largest 2

rest (**9** . (4 -1 2 8 1 2 4))

.

(6 1 2 -3 9 4 -1 2 8 1 2 4)

largest 9

2nd-largest 6

rest (**4** . (-1 2 8 1 2 4))

.

We could write a 'cond' or an 'if' expression to handle the three cases:

```
(define (2nd-max-tr largest 2nd-largest lon)
  (cond ((null? lon) 2nd-largest)
        (( case #1 ) ...)
        (( case #2 ) ...)
        (( case #3 ) ...) ))
```

The ...'s are ugly and repeat (rest lon) three times.

.

Instead, let's use our interface procedure as inspiration:

```
(define (2nd-max-tr largest 2nd-largest lon)
  (if (null? lon)
      2nd-largest
      (2nd-max-tr
        new value of largest
        new value of second
        new value of lon      ) )))
```

.

Handle (first lon) in first two arguments:

```
(define (2nd-max-tr largest 2nd-largest lon)
  (if (null? lon)
      2nd-largest
      (2nd-max-tr
        new value of largest
        new value of second
        new value of lon      )))
```

•

Handle (rest lon) in the third argument:

```
(define (2nd-max-tr largest 2nd-largest lon)
  (if (null? lon)
      2nd-largest
      (2nd-max-tr
        new value of largest
        new value of second
        new value of lon      )))
```

•

```
(define (2nd-max-tr largest 2nd-largest lon)
  (if (null? lon)
      2nd-largest
      (2nd-max-tr
       (max largest
            (first lon))
       (max 2nd-largest
            (min largest (first lon)))
       (rest lon))))
```

This is order $O(n)$, makes only **one** pass,
and uses only one stack frame.

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How might we compare these solutions?

- length of the code
- space used at run-time
- time used at run-time
- time to create the program
- ...
- complexity of the code
- ...
- familiarity

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Now, we use all three to:

- learn how programming languages work

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Static Properties of Variables

A property is **static** when its value can be determined by looking at the text of a program.

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Static Properties of Variables

A property is **static** when its value can be determined by looking at the text of a program.

A property is **dynamic** when the program must be executed in order to determine its value.

.

example: data type

```
int sumOfSquares(int m, int n)
{
    return m*m + n*n;
}
```

```
def sum_of_squares(m, n):
    return m*m + n*n
```

•

A Little Language

```
<exp> ::= <varref>  
        | (lambda (<var>) <exp>)  
        | (<exp> <exp>)
```

•

free and **bound** variables

```
int sumOfSquares( int m, int n )  
{  
    // m and n are bound  
    // to formal parameters  
  
    return m*m + n*n;  
}
```

.

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A variable **is bound** or **occurs bound** in an expression if it refers to the formal parameter in the expression.

A variable **is free** or **occurs free** in an expression if it is not bound.

.

$\langle \text{exp} \rangle ::= \langle \text{varref} \rangle$
 $| (\text{lambda } (\langle \text{var} \rangle) \langle \text{exp} \rangle)$
 $| (\langle \text{exp} \rangle \langle \text{exp} \rangle)$

Free and bound variables in this language:

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A function definition with no free variables is called a **combinator**.

```
(lambda (f)                ; combinator
  (lambda (x)
    (f (f x)))))
```

```
(lambda (f)                ; not a combinator
  (lambda (x)
    (f (g x)))))
```

•

This is **not** a combinator:

```
(define sum-of-applications  
  (lambda (f x y)  
    (+ (f x) (f y))))
```

•