Propositional Logic as a KR

Vocabulary

symbols

connectives

Structure

A symbol is a legal sentence.

If p is a sentence, then (not p) is a legal sentence.

If p and q are sentences, then:

(p and q)
(p or q)
(p implies q)

are legal sentences.

A Truth Table for Propositional Logic

We can define the semantics of a compound logical expression using a truth table that specifies the value of the compound for each possble combination of its parts.

p	q	not p	not q	p or q	p and q	p implies q
T	T	F	F	T	T	T
T	F	F	T	T	F	F
F	T	T	F	T	F	T
F	F	T	T	F	F	T

If a compound has **n** distinct components, then it will have 2^n rows in its truth table.

Propositional Logic as a KR

Manipulation

Several different rules for inferring new sentences exist:

(p and q)	implies	p
p	implies	(p or q)
p, (p implies q)	implies	q

These rules are called inference rules, because they allow us to infer new sentences from existing sentences.

They are like our operators in search: they allow us to “change the state” of our knowledge.

Meaning

The meaning of a compound sentence is based on the meanings of its parts. The truth table tells us the meanings for connectives based on convention.

The meaning of individual propositions is based on some external semantics, say, procedural or descriptive.

Predicate Logic as a KR

Let's start with a simple sentence:

"Marcus is a man."

Representing sentences of this sort as propositions quickly leads to a **big** problem:

marcus-is-a-man
caesar-is-a-man
eugene-is-a-man
...

Objects

Marcus, Caesar, Eugene, ...

Predicates

man		man(Marcus), man(Eugene), ...

Variables

man(**x**)

Predicate Logic as a KR

Compound sentences work the same way!

not(man(Cleopatra))

man(Marcus) **and** hungry(Marcus)

man(Cleopatra) **or** woman(Cleopatra)

man(Marcus) **implies** person(Marcus)

Quantifiers

We need some way to reason about expressions with variables, in particular about what a variable means. The result is two new connectives:

∀**x** man(x) **implies** person(x)

э**x** man(x) **implies** hungry(x)

The semantics of these connectives are best understood descriptively or mathematically...

An Exercise in Using Logic to Represent Knowledge of the World

Represent the following sentences in predicate logic.

1. Eugene is a Hoosier.

2. Hoosiers like basketball.

3. Children of basketball fans are basketball fans.

4. Basketball fans like the month of March.

5. Sarah is Eugene’s daughter.

A Possible Solution to the Exercise

Remember:

- Not all statements we make about the world are true.
- The "name" of a symbol does not matter in reasoning with the symbol. Only the semantics attached to it matter.

1. Eugene is a Hoosier.

hoosier(Eugene)

2. Hoosiers like basketball.

$\forall$x hoosier(x) -> likes(x, Basketball)

3. Children of basketball fans are basketball fans.

$\forall$x, y childOf(x, y) and likes(y, Basketball)
-> likes(x, Basketball)

4. Basketball fans like the month of March.

$\forall$x likes(x, Basketball) -> likes(x, March)

5. Sarah is Eugene's daughter.

daughterOf(Sarah, Eugene)

Rules of Inference

Two common rules for reasoning over predicate logic representations rely on a simple rule from propositional logic involving implication, extended for reasoning with variables:

Modus Ponens

man(Marcus)	p
<u>$\forall$x man(x) **implies** person(x)</u>	<u>p -> q</u>
person(Marcus)	q

Modus Tolens

not(person(Lassie))	not q
<u>$\forall$x man(x) **implies** person(x)</u>	<u>p -> q</u>
not(man(Lassie))	not p

An Exercise in Using Logic to Reason

From your representation of:

1. Eugene is a Hoosier.
2. Hoosiers like basketball.
3. Children of basketball fans are basketball fans.
4. Basketball fans like the month of March.
5. Sarah is Eugene's daughter.

CONCLUDE: Sarah likes March.

1. **hoosier(Eugene)**

2. $\forall$**x hoosier(x) -> likes(x, Basketball)**

3. $\forall$**x, y childOf(x, y) and likes(y, Basketball)**
   -> likes(x, Basketball)

4. $\forall$**x likes(x, Basketball) -> likes(x, March)**

5. **daughterOf(Sarah, Eugene)**

An Exercise in Using Logic to Reason

GIVEN:

1. **hoosier(Eugene)**
2. ∀**x hoosier(x) -> likes(x, Basketball)**
3. ∀**x, y childOf(x, y) and likes(y, Basketball)**
   -> likes(x, Basketball)
4. ∀**x likes(x, Basketball) -> likes(x, March)**
5. **daughterOf(Sarah, Eugene)**

CONCLUDE: **likes(Sarah, March)**

Oops! Rule 3 refers to a child-of relationship, and Rule 5 refers to a daughter-of relationship, but I don't represent the **commonsense knowledge** that a daughter is also a child!

6. ∀**x, y daughterOf(x, y)-> childOf(x, y)**

7. **likes(Eugene, Basketball)** [1,2]

8. **childOf(Sarah, Eugene)** [5,6]

9. **likes(Sarah, Basketball)** [3,7,8]

10. **likes(Sarah, March)** [4,9]

Evaluating a Knowledge Representation

We evaluate a search strategy using several criteria:

- completeness Does it guarantee a solution if one exists?
- time How long does it take?
- space How much storage is required?
- optimality Does it find the "best" solution if there are many?

We can apply these rules to KRs, too.

How do prepositional and predicate logics measure up?

An Exercise: Logic as Representation

Represent the following sentences in logic.

1. One more outburst like that and you'll be in contempt of court!
2. A new *Friends* is on TV tonight, if you're interested.
3. Either the Reds win the pennant, or I'm out $10.00.
4. Maybe I'll come to your party, and maybe I won't.
5. Well, I like Bob and I don't like Bob.

1. One more outburst like that and you'll be in contempt of court!

   Straightforward translation:
   haveOutburst(You) and inContempt(You)

   Consequence:
   inContempt(You) must be true!

   Intended meaning:
   haveOutburst(You) ***implies*** **inContempt(You)**

More Answers

2. *Friends* is on TV tonight, if you're interested.

   Straightforward translation:
   interested(You, *Friends*)
   implies onTubeTonight(*Friends*)

   Consequence:
   What if I am **not** interested?

   Intended meaning:
   onTubeTonight(*Friends*) and
   (interested(You, *Friends*)
   implies canWatchTonight(You,*Friends*))

3. Either the Reds win the pennant, or I'm out $10.00.

   Straightforward translation:
   win(Reds, pennant-of(NL, 1999)) or
   lose(EW, $10)

   Consequence: What if the Reds win?
   Where is the *causality*?

   Intended meaning:
   prevent(win(Reds, pennant-of(NL, 1999)),
   lose(EW, $10))

More Answers...

4. Maybe I'll come to your party, and maybe I won't.

   Straightforward translation:

   (maybe(comeTo(EW, YourParty))) or
   (maybe(not comeTo(EW, YourParty)))

   Consequence: The translation is a tautology!

   Intended meaning:

   undecided comeTo(EW, YourParty)

   Consequence:

   We need an operator that deals with *possibility*...

5. Well, I like Bob and I don't like Bob.

   Straightforward translation:

   like(EW, Bob) and not like(EW, Bob)

   Consequence: The translation is a contradiction!

   Intended meaning:

   ∃ way1, way2
   like(EW, Bob, way1) and
   not like(EW, Bob, way2))

   Consequence: We need to represent states explicitly.