

# PHYSICS 4750 Physics of Modern Materials

## Chapter 8: Magnetic Materials

### 1. Atomic Magnetic Dipole Moments

A magnetic solid is one in which at least some of the atoms have a permanent magnetic dipole moment  $\vec{\mu}$ . The atoms are said to be *moment-bearing*. The magnetic dipole moment for a single electron orbiting a nucleus can be written as

$$\vec{\mu} = -\frac{\mu_B}{\hbar}(\vec{l} + g\vec{s}), \quad (8.1)$$

where  $\mu_B = e\hbar/2m_e$  is the Bohr magneton,  $\vec{l}$  is the (quantized) orbital angular momentum of a single electron,  $\vec{s}$  is the spin angular momentum, and  $g$  is the electron  $g$  factor  $\cong 2$ . If a magnetic field  $\vec{B}$  is applied in the  $z$  direction and a magnetic dipole is immersed in it, the moment will experience a torque and an attendant potential energy of orientation given by

$$E = -\vec{\mu} \cdot \vec{B} = -\mu_z B. \quad (8.2)$$

For a single electron, this becomes

$$E = -\mu_z B = \frac{\mu_B B}{\hbar}(l_z + g s_z) = \mu_B B(m_l + 2m_s). \quad (8.3)$$

In most magnetic atoms, two or more electrons (usually  $d$  or  $f$  electrons) contribute to the total moment of the atom. Assuming all other electrons ( $s$  and  $p$ ) are paired up, the total angular momentum  $\vec{J}$  for the entire atom is then given by

$$\vec{J} = \vec{L} + \vec{S}, \quad (8.4)$$

where  $\vec{L}$  is the total orbital angular momentum of the electrons that contribute to the moment and  $\vec{S}$  is their total spin angular momentum. Such an atom when immersed in a field has energy

$$E = -\mu_z B = g_J \mu_B J_z B / \hbar = g_J \mu_B M_J B, \quad (8.5)$$

where  $M_J$  can have any integer value in the range  $-J \leq M_J \leq J$  and  $g_J$  is called the Landé  $g$  factor. In the preceding,  $J$  is the total angular momentum quantum number.

### 2. Magnetization and Magnetic Susceptibility

The magnetization  $\vec{M}$  of a sample of material is defined as the total magnetic moment per unit volume. For a homogeneous medium,  $\vec{M}$  is the same everywhere in the sample. Of course, at the edges where the homogeneity is lost, the magnetization will differ from the bulk value.

We mentioned the magnetic field or magnetic flux density  $\vec{B}$  above. The unit of  $\vec{B}$  is the tesla (T). In vacuum (free space), this field is related to another field  $\vec{H}$ , which is also called the magnetic field:

$$\vec{B} = \mu_0 \vec{H}. \quad (8.6)$$

In Eq. (8.6),  $\mu_0 = 4\pi \times 10^{-7} \text{ Hm}^{-1}$  is the permeability of free space.  $\vec{H}$  is an applied magnetic field resulting from a current or a magnet.  $\vec{B}$  is a response to the applied field, either in free

space or a material medium. If there is a material present in the applied magnetic field, the relationship becomes

$$\vec{B} = \mu_0(\vec{H} + \vec{M}). \quad (8.7)$$

For a linear material, the magnetization is proportional to  $\vec{H}$ . The proportionality constant is called the magnetic susceptibility:

$$\vec{M} = \chi\vec{H}. \quad (8.8)$$

In SI units, both  $\vec{M}$  and  $\vec{H}$  are measured in units of  $\text{Am}^{-1}$ , so the susceptibility is dimensionless. Substituting Eq. (8.8) into Eq. (8.7) gives

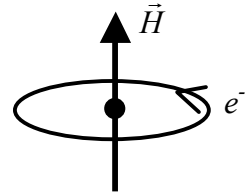
$$\vec{B} = \mu_0(1 + \chi)\vec{H} = \mu_0\mu_r\vec{H}, \quad (8.9)$$

where  $\mu_r = 1 + \chi$  is the relative permeability of the material.

If the susceptibility is negative, the material is said to be *diamagnetic*. The change in the magnetization of a diamagnetic material is in the opposite direction to the change in the field. Paramagnetic, ferromagnetic, and antiferromagnetic (and other categories as well!) materials have positive susceptibilities. It should be noted that all atoms and therefore all materials have a diamagnetic contribution to the total magnetization. However, the effect is very small and is overwhelmed in a paramagnetic, ferromagnetic, or antiferromagnetic material.

### 3. Diamagnetism

Experimentally, diamagnetism can be observed by placing a diamagnetic material in a strong magnetic field gradient. The material will experience a force toward the *weaker region* of the field. (See Fig. 4.1 in Spaldin.) We will derive an expression for the diamagnetic susceptibility of a material using classical means. The same result is obtained using the correct (but more complicated) quantum-mechanical method. In fact, diamagnetism is a quantum-mechanical effect. The use of the classical method gives us a way to think about diamagnetism, but it is really not correct.



Consider an electron orbiting in a plane perpendicular to an applied magnetic field. The current is in the opposite direction to the electron's velocity. If the field has been increased slowly from zero, Faraday's law tells us that there will be an induced emf:

$$emf = \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi}{dt}. \quad (8.10)$$

The induced emf will arise from the non-electrostatic electric field, which will act to oppose the flux change. Assuming the electron orbit is circular, we have

$$emf = E2\pi r = -\frac{d\Phi}{dt}. \quad (8.11)$$

The opposing of the flux change is accomplished by increasing the electron's speed (for the case pictured above), which also increases its magnetic moment ( $\mu = IA$ ). [When the field stops changing, the new value of the current (and moment) remains because there is no agent to restore the original value.]

Recall that torque  $\vec{\tau} = \vec{r} \times \vec{F} = d\vec{L}/dt$ . The torque exerted on the electron by the induced electric field is given by

$$\frac{dL}{dt} = -eEr = \frac{e}{2\pi} \frac{d\Phi}{dt} = \frac{er^2\mu_0}{2} \frac{dH}{dt}. \quad (8.12)$$

Integrating over time, the change in angular momentum is given by

$$\Delta L = \frac{er^2\mu_0}{2} H, \quad (8.13)$$

where the change in field is  $H$ . The magnetic moment of the orbiting electron can be written as

$$\mu = IA = -\frac{eL}{2m_e}. \quad (8.14)$$

Thus, the change in the magnetic moment due to the change in applied field is given by

$$\Delta\mu = -\frac{e\Delta L}{2m_e} = -\frac{e^2r^2\mu_0}{4m_e} H. \quad (8.15)$$

Hence, the induced moment is proportional to the field and in the opposite direction to it. Now, we have assumed the orbit is perpendicular to the field. Because the electron orbits are not in general perpendicular to the field, we need to take the average of the square of the radius of the projection of the orbit onto the field direction (i.e., the mean square radius). This introduces a factor of 2/3. In addition, we need to multiply by a factor of  $Z$  since there are  $Z$  electrons. Finally, we need to take the average value of all occupied orbital radii. Thus,

$$\Delta\bar{\mu} = -\frac{Ze^2 \langle r^2 \rangle_{av} \mu_0}{6m_e} \vec{H}. \quad (8.16)$$

To obtain the magnetization, we need to multiply the magnetic moment per atom by the number of atoms per unit volume  $N$ :

$$\vec{M} = N\bar{\mu} = N\Delta\bar{\mu} = -\frac{NZe^2 \langle r^2 \rangle_{av} \mu_0}{6m_e} \vec{H}. \quad (8.17)$$

The susceptibility is therefore given by

$$\chi = \frac{\vec{M}}{\vec{H}} = -\frac{NZe^2 \langle r^2 \rangle_{av} \mu_0}{6m_e}. \quad (8.18)$$

The susceptibility is always negative. Note that there is no explicit temperature dependence, but  $\langle r^2 \rangle_{av}$  is weakly temperature dependent. Typically,  $\chi \sim -10^{-6}$ , which is very small. (Show video of diamagnetic levitation of frog.)

#### 4. Paramagnetism

In diamagnetic materials, the atoms or molecules that constitute them have no permanent magnetic moment; the moment is induced upon application of a magnetic field. In contrast, the constituent atoms or molecules of a paramagnetic material have a permanent moment. The

moments do not interact with each other strongly via dipole-dipole magnetic forces because the moments are not close enough for that weak interaction to be significant. Thus, at all temperatures, the moments are randomly oriented in the absence of a magnetic field. If a magnetic field is applied, there is a slight alignment of the moments in the same direction as the field. [Fig. 5.1, Spaldin] This is rather like an electric current, in which the individual electrons are moving very rapidly and scatter frequently in different directions but there is an overall slow drift in one direction. Because the alignment is in the same direction as the field, the induced magnetization is in the same direction as field. Thus, the susceptibility  $\chi = \vec{M} / \vec{H}$  is *positive*.

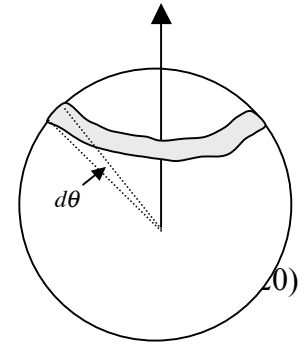
To describe the paramagnetic susceptibility quantitatively, we shall use a classical theory that is due to Langevin. The energy of interaction of a magnetic moment with an applied magnetic field is given by Eq. (8.2):  $E = -\vec{\mu} \cdot \vec{B} = -\mu_z B$ , assuming that the field is along the  $z$  axis. If the angle between the field and the direction of the moment is  $\theta$ , then  $E = -\mu B \cos \theta$ . According to Boltzmann statistics, the probability that a state with energy  $E$  is occupied is given by

$$p_{\text{Bol}} = e^{-E/k_B T} = e^{\mu B \cos \theta / k_B T}. \quad (8.19)$$

We assume that the magnitude of a moment is constant; only its direction changes. Thus, the tip of a moment can lie anywhere on the surface of a sphere whose radius is equal to the magnitude of the moment. The number of moments that make angles that lie between  $\theta$  and  $\theta + d\theta$  is proportional to the area of a ring of thickness  $rd\theta$ , where  $r$  is the radius of the sphere. This area is

$dA = (2\pi r \sin \theta)(rd\theta) = 2\pi r^2 \sin \theta d\theta$ . It follows that the probability that a moment has a direction such that it makes an angle with the  $z$  axis that lies between  $\theta$  and  $\theta + d\theta$  is

$$p(\theta, T, B) = \frac{p_{\text{Bol}} dA}{\int_{\text{sphere}} p_{\text{Bol}} dA} = \frac{e^{\mu B \cos \theta / k_B T} \sin \theta d\theta}{\int_0^\pi e^{\mu B \cos \theta / k_B T} \sin \theta d\theta}. \quad (8.20)$$



The magnetization is the average  $z$ -component of the total magnetic moment per unit volume, i.e., the average value of the component of the total moment per unit volume along the direction of the field. For a given moment, the component along the direction of the field (i.e.,  $z$  component) is  $\mu \cos \theta$ . Thus, the magnetization is

$$M = \int N \mu \cos \theta p(\theta, T, B) = N \mu \frac{\int_0^\pi e^{\mu B \cos \theta / k_B T} \cos \theta \sin \theta d\theta}{\int_0^\pi e^{\mu B \cos \theta / k_B T} \sin \theta d\theta}, \quad (8.21)$$

where  $N$  is the number of moments per unit volume. Performing the integrations yields

$$M = N \mu \left[ \coth \left( \frac{\mu B}{k_B T} \right) - \frac{k_B T}{\mu B} \right] = N \mu L(x), \quad (8.22)$$

where  $x = \mu B / k_B T$  and  $L(x) = \coth(x) - 1/x$  is the Langevin function. For  $B \rightarrow \infty$  or  $T \rightarrow 0$ ,  $L(x) \rightarrow 1$ , which is the maximum value of the Langevin function. Thus, the maximum magnetization of the paramagnet is  $N\mu$ , i.e., the magnetization when all the moments are aligned with the field.

The Langevin function can be expanded as a Taylor series:

$$L(x) = \frac{x}{3} - \frac{x^3}{45} + \dots \quad (8.23)$$

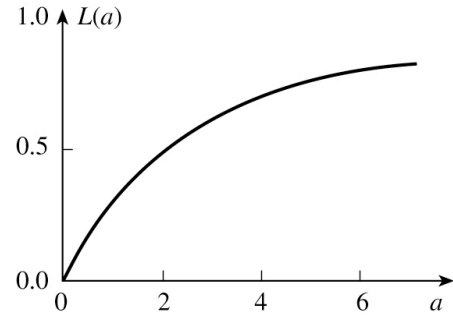
If  $x$  is small, (i.e., small  $B$  or high  $T$ ), then  $L(x) \approx x/3$ . In this regime, the magnetization becomes

$$M = \frac{N\mu x}{3} = \frac{N\mu^2 B}{3k_B T}. \quad (\text{Small } B, \text{ high } T) \quad (8.24)$$

The magnetic susceptibility is therefore given by

$$\chi = \frac{M}{H} = \frac{M}{B/\mu_0} = \frac{\mu_0 N \mu^2}{3k_B T}. \quad (8.25)$$

(Recall that  $B = \mu_0 H$ , where  $\mu_0$  is the permeability of free space, which is unrelated to the magnitude  $\mu$  of the magnetic moment.) We see that in the small-field, high-temperature regime, the susceptibility is independent of the field and is inversely proportional to the temperature. Eq. (8.25) is called the Curie law. The constant of proportionality  $C = \mu_0 N \mu^2 / 3k_B$  is called the Curie constant.



The Langevin function form for the magnetization of a paramagnet was derived using classical considerations, i.e, the moment could point in any direction. This is a good approximation if the moments are large, as in *superparamagnets*, which are materials in which the moments are supermoments consisting of many elementary moments that are all aligned. However, for most paramagnetic materials, the moments are those of single atoms or molecules, whose  $z$ -components are quantized because of the quantization of angular momentum. Thus, to calculate the magnetization, one has to *sum* over the finite number of quantized orientations (instead of *integrating* over the continuous variable  $\theta$ ). Consider an individual magnetic moment which has a  $z$  component given by  $\mu_z = -g_J \mu_B M_J$ , where  $-J \leq M_J \leq J$ . Performing the appropriate summation over all possible  $M_J$  values yields the following equation for the magnetization:

$$M = Ng_J \mu_B J \left[ \frac{2J+1}{2J} \coth\left(\frac{2J+1}{2J} y\right) - \frac{1}{2J} \coth\left(\frac{y}{2J}\right) \right] = Ng_J \mu_B J B_J(y), \quad (8.26)$$

where  $y = g_J \mu_B JB / k_B T$  and  $B_J(y)$  is called the Brillouin function. It is gratifying that when one takes the limit  $J \rightarrow \infty$ , the Langevin function is recovered.

The Brillouin function has the following Taylor expansion:

$$B_J(y) = \frac{J+1}{3J} y - \frac{[(J+1)^2 + J^2](J+1)}{90J^3} y^3 + \dots \quad (8.27)$$

For small  $y$  (small  $B$ , high  $T$ ), we see that  $B_J(y) \approx (J+1)y/3J$ . Thus, for small  $y$ ,

$$\chi = \frac{\mu_0 N g_J^2 \mu_B^2 J(J+1)}{3k_B T} = \frac{C}{T}, \quad (\text{Small } B, \text{ high } T) \quad (8.28)$$

which is the Curie law for quantum-mechanical magnetic moments. [Show MATLAB graphs for Langevin and Brillouin functions on the same axes.]

In deriving the Curie law for the paramagnetic susceptibility, we assumed that the magnetic moments were essentially isolated, i.e., interactions were negligible. In many paramagnets, the simple Curie law is not followed because interaction between moments is not negligible. Instead, the Curie-Weiss law is valid:

$$\chi = \frac{C}{T - \Theta}, \quad (8.29)$$

where  $C$  is the same Curie constant that we saw before and  $\Theta$  is called the Weiss temperature, which is a measure of the strength of the interaction between moments. Let us derive the Curie-Weiss law using a classical argument due to Weiss.

To model the interaction between the atomic moments, Weiss introduced the concept of the *molecular field*. Each moment will experience an effective field that is a superposition of the effects of its interactions with all the other moments in the material. This effective field has the same effect as an applied magnetic field and Weiss assumed that this field is proportional to the magnetization of the material. (This makes sense: stronger moments should lead to stronger interactions.) Thus, the net field experienced by a moment is due to the applied field  $H$  and the molecular field  $H_m$ , where

$$H_m = \lambda M. \quad (8.30)$$

Hence,

$$\frac{M}{H_{tot}} = \frac{M}{H + H_m} = \frac{M}{H + \lambda M} = \frac{C}{T}. \quad (8.31)$$

Using the last two expressions in Eq. (8.31) to solve for  $M$  gives

$$M = \frac{CH}{T - \lambda C}. \quad (8.32)$$

Hence,

$$\chi = \frac{M}{H} = \frac{C}{T - \Theta}, \quad (8.33)$$

where  $\Theta = \lambda C$ .

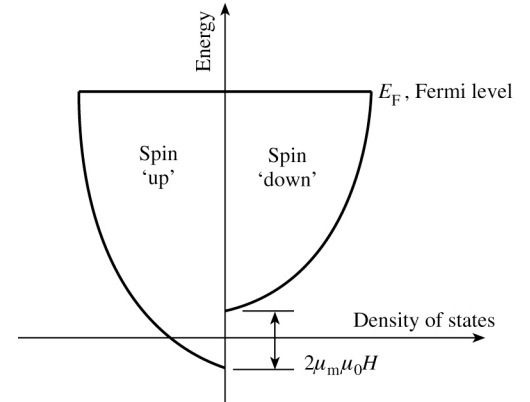
Note that the susceptibility diverges as the temperature approaches  $\Theta$ . This divergence of the susceptibility signals a phase transition to a state in which the magnetic moments are spontaneously ordered at low enough temperatures. Ferromagnetism is such a state. In ferromagnetism, there is a spontaneous ordering of the moments such that neighboring moments align themselves parallel to each other over macroscopic regions. In Weiss's theory, the interaction that causes this parallel alignment is the molecular field. As we shall see when we discuss ferromagnetism in more detail, molecular field theory leads to a paramagnetic to ferromagnetic phase transition temperature  $T_C = \Theta$ . At temperatures greater than  $T_C$ , the thermal energy dominates the molecular-field energy and the system is paramagnetic. At temperatures below  $T_C$ , the molecular-field energy exceeds the thermal energy and the ordered ferromagnetic

state exists. Thus, at  $T_C$ , the molecular field energy and the thermal energy are approximately equal. This gives a method of estimating the strength of the molecular field:  $k_B T_C \approx \mu_0 \mu_B H_m$ . So,  $\mu_0 H_m \approx k_B T_C / \mu_B$ . Using  $T_C \approx 1000$  K (close to that for iron), we find that  $\mu_0 H_m \approx 10^3$  T, which is an extremely large field. (Typically, the largest fields that are produced in labs are  $\sim 10$  T.) It turns out that this strong interaction between elementary magnetic moments is quantum mechanical in origin. It is called the *exchange interaction*, and arises because of the overlap of electron wave functions and the Pauli principle.

We have seen that magnetic moments that interact weakly give rise a temperature-dependent susceptibility (Curie-Weiss law). Some metals exhibit a paramagnetic susceptibility that is temperature-independent. It is called Pauli paramagnetism. As we have seen before, the electrons close to the Fermi level of a metal give rise to the temperature-dependent properties. This is true in the case of the magnetic susceptibility, but it turns out the temperature dependence cancels out.

Each conduction electron in a metal (like all the other electrons) has a permanent spin magnetic moment of magnitude  $\mu_B$ . The  $z$ -component of the magnetic moment has only two possible orientations – approximately parallel to an external field and approximately antiparallel to it (spin down or spin up). In the presence of an applied magnetic field, more electrons will have their moments parallel to the field than antiparallel to it and so there will be a magnetization. Let us calculate this magnetization and the magnetic susceptibility of a metal, which is represented by a free-electron gas.

The figure shows the density of states for the parallel and antiparallel electron moments. The temperature is low, so that there are few occupied states above the Fermi level, and nearly all the states below the Fermi energy are occupied. When the field is first turned on, the electron energies change and the electron distribution adjusts so that the Fermi levels of spin up and spin down electrons are equal. The magnetization of the electron gas can therefore be calculated as



$$M = \frac{1}{2} \int_{-\mu_B B}^{E_F} \mu_B F(E) Z(E + \mu_B B) dE - \frac{1}{2} \int_{\mu_B B}^{E_F} \mu_B F(E) Z(E - \mu_B B) dE. \quad (8.34)$$

The factor of  $\frac{1}{2}$  accounts for the fact that we have separate densities of states for spin up and spin down electrons and so we have to eliminate the twofold degeneracy for spin. We take  $F(E) = 1$  and make substitutions of variables ( $E' = E + \mu_B B$  and  $E'' = E - \mu_B B$ ) to rewrite the integrals:

$$M = \frac{1}{2} \mu_B \left[ \int_0^{E_F + \mu_B B} Z(E') dE' - \int_0^{E_F - \mu_B B} Z(E'') dE'' \right] = \frac{1}{2} \mu_B \int_{E_F - \mu_B B}^{E_F + \mu_B B} Z(E) dE. \quad (8.35)$$

Since  $\mu_B B \ll E_F$ ,

$$\int_{E_F - \mu_B B}^{E_F + \mu_B B} Z(E) dE \approx Z(E_F) \Delta E = 2 \mu_B B Z(E_F). \quad (8.36)$$

Thus,

$$M = \mu_B^2 Z(E_F) B = \mu_0 \mu_B^2 Z(E_F) H, \quad (8.37)$$

and so

$$\chi = \frac{M}{H} = \mu_0 \mu_B^2 Z(E_F). \quad (8.38)$$

We see that the susceptibility is indeed temperature-independent. In a real metal, the susceptibility is measured to be smaller than the value in Eq. (8.38) because of a diamagnetic contribution.

## 5. Ferromagnetism

As we have seen previously, in the classical Weiss theory, the elementary magnetic moments interact via the molecular field, which gives them a tendency to align parallel to each other. At a low enough temperature, the aligning influence of the molecular field overcomes the disordering influence of thermal energy and the material becomes spontaneously ordered, i.e., there is parallel alignment of the moments in the absence of an external applied field over macroscopically-sized regions. This ordered state with parallel moments constitutes ferromagnetism.

Let us assume that the ferromagnetic state can be described by the Langevin function, with the molecular field added to the applied field. (The Brillouin function would be more accurate, but it is a bit more complex.) Then

$$M = N\mu L \left[ \frac{\mu_0 \mu (H + H_m)}{k_B T} \right]. \quad (8.39)$$

This gives

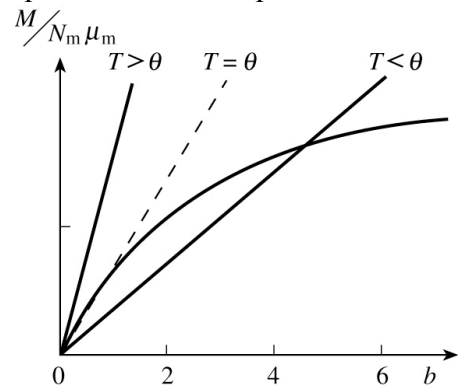
$$\frac{M}{N\mu} = L \left[ \frac{\mu_0 \mu (H + \lambda M)}{k_B T} \right]. \quad (8.40)$$

Let us look for a solution that gives a non-zero magnetization when the applied field  $H$  is zero. This is called the spontaneous magnetization and is characteristic of the ferromagnetic state. Thus, Eq. (8.40) becomes

$$\frac{M}{N\mu} = L \left[ \frac{\mu_0 \mu \lambda M}{k_B T} \right]. \quad (8.41)$$

A useful way to find the solution is to graph both sides of the equation and find a point of intersection. The figure shows this for various values of  $T$ . (Note that in the figure,  $b = \mu_0 \mu \lambda M / k_B T$ . Thus, the LHS of Eq. (8.41) is  $M / N\mu = (k_B T / N\mu_0 \mu^2 \lambda) b$ . Hence, the slope of the straight line representing the LHS is proportional to  $T$ .) For high temperatures, there is no point of intersection (excluding the origin). This is the paramagnetic state. At a particular temperature  $T = \Theta$ , the straight line (LHS) is tangent to the Langevin function (RHS) at the origin. For temperatures  $T > \Theta$ , there is a single point of intersection corresponding to a non-zero spontaneous magnetization.

Thus, the temperature  $T = \Theta$  is the transition temperature where the paramagnetic to





ferromagnetic phase transition takes place. The transition temperature is also called the Curie temperature  $T_C$ .

We can determine the transition temperature by equating the slope of the straight line represented by the LHS of Eq. (8.41) to that of the Langevin function (representing the RHS of that equation) at the origin. Near the origin ( $b = 0$ ),  $L(b) \approx b/3$  as seen before. Thus, the slope of the Langevin function at the origin is  $1/3$ . The slope of the straight line is  $k_B T / N \mu_0 \mu^2 \lambda$  as seen above.

Equating the slopes gives

$$\frac{k_B T_C}{N \mu_0 \mu^2 \lambda} = \frac{1}{3}, \quad (8.42)$$

or,

$$T_C = \frac{N \mu_0 \mu^2 \lambda}{3 k_B}. \quad (8.43)$$

Thus, the Curie temperature is proportional to the molecular field constant  $\lambda$ . This makes sense: the stronger the interactions between the moments, the greater the thermal energy required to destroy the order.

As alluded to above, the use of the Brillouin function gives a better description of the magnetization of real ferromagnetic materials. [Fig. 6.3 in Spaldin] This is not unexpected, since the elementary moments in common ferromagnetic materials are atomic moments that must be described quantum mechanically.

The Brillouin function would predict that the saturation magnetization of a ferromagnetic material should always be an integer number of Bohr magnetons per magnetic ion (i.e.,  $g \mu_B J = 2 J \mu_B$ ). However, for transition-element ferromagnets such as Fe, Co, and Ni, one finds that the saturation magnetization per ion is non-integral. The reason is that the ferromagnetism arises from *collective* behavior of the electrons that give rise to the magnetism. In the Weiss theory, the magnetic ions are treated as *local moments*, i.e., the magnetic moment is due to electrons that are localized to the parent atom.

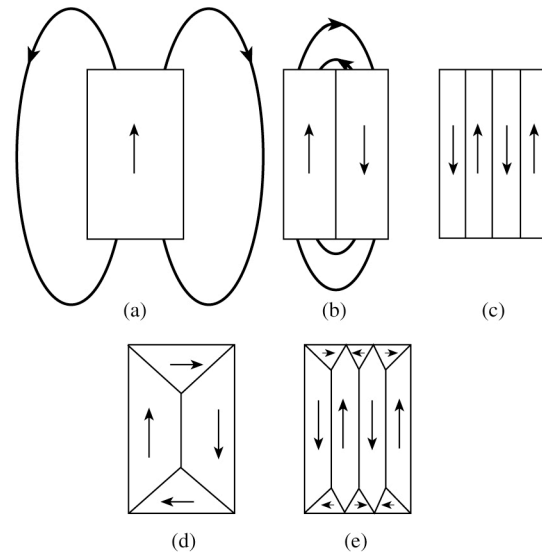
In collective electron ferromagnetism, the exchange interaction plays a similar role to the external magnetic field in Pauli paramagnetism. It lowers the energy of one spin direction (let us say spin up) and raises the energy of the other spin direction (spin down). Thus, it might seem that to minimize the total energy, the electrons that are favorably aligned (spin up) should outnumber those that are not (spin down). However, since the electron states are in a band, the spin down electrons are moved up in energy in their sub-band and if the density of states is low, the energy cost becomes prohibitive. Thus, the numbers of spin up and spin down electrons remains about the same. If, however, the density of states at the Fermi level is high, e.g., for the  $d$  band electrons in transition metals, a relatively large number of spin down electrons can move up in energy, but with a relatively small energy change. (Remember: a high density of states means a large number of states per unit energy interval.) Thus, for a high-enough exchange energy and a high-enough density of states at the Fermi level, there can be a significant difference in the population of spin up and spin down electrons in the minimum-energy configuration. Thus, there is a macroscopic magnetization without the presence of an external field, i.e., ferromagnetism. Because the magnetization involves differing numbers of electrons occupying band states, the

magnetic moment per atom need not be an integral number of Bohr magnetons. [Insert Fig. 6.6, Spaldin.]

### 5a. Domains

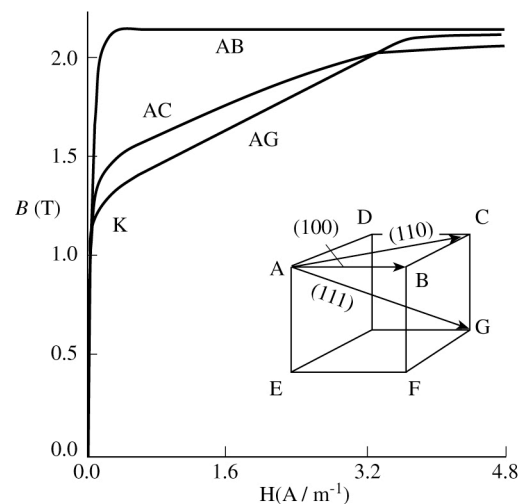
A paper clip is made of (mostly) iron, which is ferromagnetic at room temperature. Why doesn't a paper clip behave like a magnet? The reason is that the magnetization of the paper clip is zero because of the formation of *domains*. Domains are regions within a ferromagnetic material containing many atoms whose moments point in the same direction. Thus, each domain has a net magnetization. However, the magnetization (magnitude and direction) varies from domain to domain so that the total magnetization of all the domains (i.e., the entire sample) is zero in the absence of an external magnetic field.

Domains form in order to minimize the total energy of the ferromagnetic material. Consider a material consisting of a single domain (i.e., all of it is magnetized in the same direction). The magnetization changes discontinuously at the surfaces, forming free poles. These poles are sources of a magnetic field (called the *demagnetization field*) that fills all of space. The energy associated with this field, called magnetostatic or demagnetization energy, is proportional to the square of the magnetization. Thus, for a single domain with a large magnetization, the magnetostatic energy is large. This energy can be reduced by the formation of domains. Domains form in such a way that opposite poles are close to each other thereby confining the path of the flux lines and reducing the net magnetization to a value close to zero. This reduces the magnetostatic energy. Note that the moments at the boundary between domains of opposite magnetization are misaligned, which increases the exchange energy. Thus, the formation of domain boundaries, or *domain walls*, also costs some energy. We will delve more deeply into this issue when we study domain walls.



### 5b. Magnetocrystalline Anisotropy

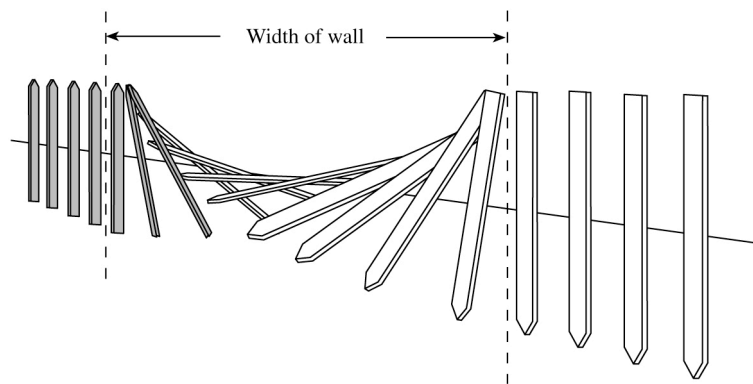
In a ferromagnetic crystal, there is a lowering of the energy if the magnetization points along certain directions. Thus, the material will magnetize more easily if an external magnetic field is directed along certain axis called the *easy axis*. (There may be more than one.) The energy associated with the alignment of the magnetization relative to crystalline axes is called the *magnetocrystalline anisotropy energy*. This



phenomenon is shown in the figure to the right, which displays the magnetization of iron versus field for three different crystallographic directions. (Fe has a cubic crystal structure.) The (100) crystallographic axes (cube edges) are the easy axes, whereas the (111) axes (body diagonals) are the hard axes. The (110) axes (face diagonals) are intermediate. To minimize magnetocrystalline anisotropy energy, domains form such that their magnetizations points along local easy axes. At a domain wall, the magnetization direction changes and so both magnetization cannot lie along an easy axis. This tends to raise the anisotropy energy. For crystals with easy axes that are perpendicular to one another (e.g., Fe), *domains of closure* can form, which give the most effective arrangement to minimize the total energy of the ferromagnetic system. [See (d) and (e) in figure above that shows domain arrangements.]

### c. Domain Walls

Domain walls are the boundaries or transition regions between domains. These walls are usually between either domains with opposite magnetizations (180° walls) or domains with magnetization that are at right angles (90° walls). We have seen that domain walls tend to increase exchange and anisotropy energy. In fact, the thickness of a domain wall



is dictated by the need to minimize the sum of the two energies. Exchange energy is minimized when the domains have parallel magnetizations. Thus, to minimize exchange energy, the domain wall should be wide so that moments within the wall are as closely aligned as possible. (The rotation is "stretched out" as much as possible to make neighboring moments as close to parallel as possible.) On the other hand, to minimize anisotropy energy, the walls should be narrow because this minimizes the number of moments within the transition region that are misaligned with the local easy axis. Thus, the width of the wall is dictated by the relative strengths of the exchange and anisotropy energies. The wall width for a cubic crystal is given by

$$\delta = \pi \sqrt{\frac{A}{K}}, \quad (8.44)$$

where  $A$  is an exchange parameter and  $K$  measures the strength of the anisotropy.

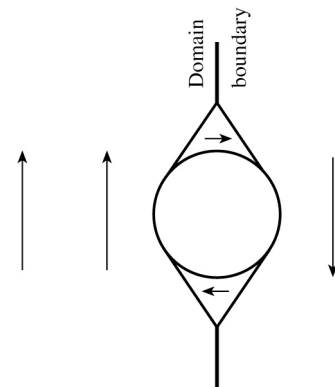
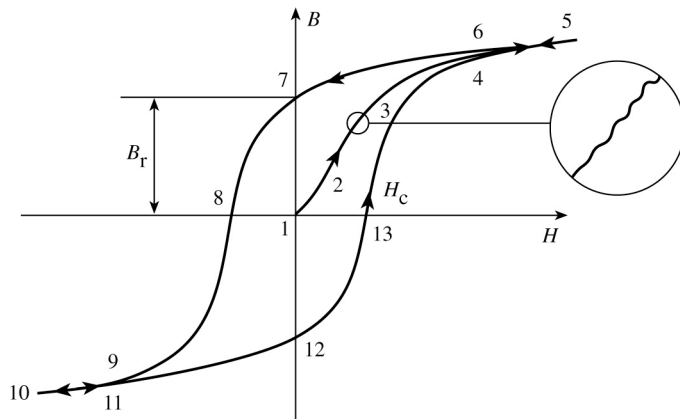
We should mention that *magnetostriction* also plays a role in the energy balance of domain walls. Magnetostriction is the change in length of a ferromagnetic material when it is magnetized. Because of magnetostriction, changes in length of a ferromagnetic material will cause changes in the magnetization, which may affect the directions of easy magnetization.

### d. Magnetization Process and Hysteresis

Let us start from a completely demagnetized sample (net magnetization = 0) and apply an increasing magnetic field starting from zero. For small fields, the magnetization increases fairly rapidly because of the movement of domain walls. The walls move easily so that domain

magnetizations that are favorably oriented with respect to the field grow, and those that are not, shrink. As the domain walls move, they may become pinned by a defect within the material. The pinning occurs because the wall lowers the magnetostatic energy associated with the defect. To further increase the magnetization, the pinned walls must be dislodged by higher magnetic fields. Eventually, all the walls are removed from the sample, which becomes a single domain. At even higher fields, the domain magnetization will rotate to align with the easy axis closest to the field direction. Saturation of the sample requires the domain magnetization to rotate away from the local easy direction to the direction of the field. If the anisotropy energy is high, this will require a large field.

If the field is now reduced, the domain magnetization will rotate back toward the nearest easy axis; this process is reversible. Eventually, domains of reverse magnetization nucleate, driven by the demagnetization field and the sample becomes partially demagnetized. However, the walls cannot be restored to their former positions because the demagnetization field is not strong enough to overcome all the energy barriers created when domain walls intersect defects. Thus, the magnetization is not reversible and the ferromagnet displays *hysteresis*. When the field is reduced to zero, the material retains a non-zero magnetization in the original direction of the field. This magnetization is called the remanent magnetization or *remanence*. The application of a field in the reverse direction is necessary to bring the magnetization back to zero. The magnitude of this field is called the *coercivity* or *coercive field*. [Fig. 7.17, Spaldin.]



If the coercivity of a ferromagnetic material is high, the material is said to be a *hard* magnetic material. These materials are useful for permanent magnets, which also usually possess a large remanent magnetization. The product of the coercivity and the remanent magnetic flux density is called the energy product, which is a figure of merit for permanent magnets. Neodymium-iron-boron (Nd-Fe-B) magnets have one of the highest energy products of all magnets. Applications of permanent magnets include electric motors, generators, and battery-free flashlights. If the coercivity of a material is low, i.e., it is easily magnetized and demagnetized, it is a *soft* magnetic material. Applications include magnetic recoding heads, transformer cores, and high-frequency devices.