Prove: If k is any odd integer and m is any even integer than k2 + m2 is odd.

PROOF:

Let k be any odd integer and m be any even integer (Given)

Thus k = 2x + 1 for some integer x (def of an odd number)

And m = 2j for some integer j (def of an even number)

We need to show that k2 + m2 is odd

k2 + m2 = (2x + 1)2 + (2j)2 (substitution)

 = 4x^2 + 4x + 1 + 4j^2 (algebra)

 = 4 ( x^2 + x + j^2 ) + 1 (algebra)

 = 2 \* 2\*(x^2 + x + j^2 ) + 1 (algebra)

Let r = 2\*(x^2 + x + j^2) (substitution)

Note r is an integer (based on closure of integers on multiplication and addition)

Thus k2 + m2 = 2 (r) + 1 (substitution)

And k2 + m2 is odd by definition of an odd number

Prove: The sum of any two rational numbers is a rational number.

PROOF:

Let x and y be rational numbers (given)

Thus x = a/b and y=c/d for integers a, b, c, and d AND b<>0 and d<>0

 [definition of rational numbers]

X + y = a/b + c/d [substitution]

Note that a/b = a\*d/b\*d [Algebra]

Note that c/d = c\*b/d\*b

X + y = a\*d/b\*d + c\*b/d\*b [substitution]

 = a\*d/b\*d + c\*b/b\*d

 = [a\*d + c\*b] / b\*d

Let m = a\*d + c\*b. Note, m is an integer by closure of integers on + and \*

Let n = b\*d. Note, n is an integer

X + y = m/n for integers m and n substitution

Therefore x+ y is a rational number by definion

Prove: For integer n if n^2 is even then n is even.

PROOF:

The contrapositive of this statement is

If n is odd then n^2 is odd.

If we can prove the contrapositive is true, then we have proven that the original statement is true.

Let n be odd

N = 2k+ 1