Passing A Serve In Different Air Densities
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I. Background
Volleyball was originally an outdoor-indoor game. Players who played the game outdoors had to deal with natural phenomena such as wet surfaces, wind, heat, cold, humidity and sun glare. When the game was brought totally indoors, most of the atmospheric variables were controlled to the point they had no effect on a player’s performance. With the consistency of indoor conditions, most scientific inquiries have focused on the technical aspects of the player's performance and their conditioning. The atmospheric variables that were influential in outdoor play are now believed to be non-consequential in indoor games. The U.S.A. National Teams have been training in Colorado Springs for the last four years. During the last Olympics the Men's National Team performed well below their potential in Sydney, Australia. One of the skills performed well below par was identified to be passing the serve. The national office requested to conduct research examining the effect varying air density on the flight of the ball during the topspin serve. Considering the speed of the topspin serve at the men’s international competition, it seemed unlikely that any considerable effect could be detected. Dr. Iradge Ahrabi-Fard, Professor and Head Volleyball coach at the University of Northern Iowa and Dr. Michael Roth, an on-campus colleague from the UNI Physics Department, were asked to conduct appropriate research into passing the serve in different air densities. Dr. Roth designed the study to investigate this concept.

The merit of this study is that its results can benefit the athletes who compete at different altitudes. If the flight of the ball changes considerably in different air densities with all other aspects of the serve and players held constant, the effect on passing this serve can be devastating. Previous investigations indicate that the passer doesn't see the ball within the last few feet of arm contact. Most of the training is reading the flight, moving to the area of passing, extending the arms early, and developing a kinesthetic feeling of predicting the area of contact before contacting the pass. A player who trains in a certain air density with a predicted ball trajectory develops certain expectations of the serve flight pattern. If this flight trajectory is different from the training condition, it may adversely affect the passer’s ability to pass accurately.

II. The Model
In modeling the flight of a volleyball it is crucial to identify the forces acting on the object so the equation describing its motion may be determined. In the simulations conducted in this study the differential equation for the ball’s motion is

\[
\frac{d\vec{v}}{dt} = -g\hat{z} - \frac{C_d}{2m} \frac{\rho \pi r^2 v^2}{v} \hat{v} + \frac{\rho \pi r^3 \omega v}{2m} \hat{\omega} \times \hat{v} + \frac{\rho}{\rho_b} g \hat{z}.
\]  

(1)

The first term is the gravitational force. Here \( g \) is the acceleration of gravity at the surface of the earth, \( m \) is the mass of the ball and \( \hat{z} \) is a unit vector in the vertical direction which, when coupled with the negative sign leading the term indicates mathematically that gravity acts downward. Typical first-semester physics courses use only this force to model projectile motion and hence afford students a solid understanding of two-dimensional kinematics.

In reality, however, such a model does not take into account the fact that the air the ball is passing through is a fluid and hence exerts both dynamic and static forces on an object passing through it. The most influential force from the air is drag resistance, expressed in the second term of Equation 1. Here \( \rho \) is the density of the air, \( r \) is the radius of the ball, \( C_D \) is the drag coefficient, \( v \) is the speed of the ball and \( \hat{v} \) is a unit vector if the direction of the ball’s velocity which, when
coupled with the negative sign in leading the term, indicates that this force always acts in opposition to the ball’s motion.

The second most important air-related force is the Magnus force, which is expressed in the third term of Equation 1 describes the lift (or drop) that a ball can obtain because it is moving through a fluid and spinning. Here \( \omega \) is the angular speed (rate of rotation) of the ball and \( \hat{\omega} \) is a unit vector in the direction of the angular velocity of the ball, which in our case may be thought of as the axis the ball is spinning about, pointing out of the pole of the ball rotating counterclockwise as viewed by an observer. The vector cross product \( \hat{\omega} \times \hat{v} \) describes this lift force as being perpendicular to both the rotational axis of the ball and the ball’s velocity. Furthermore, if the ball has underspin, \( F_L > 0 \) and the ball gains lift relative to its path without spin; if the ball has topspin \( F_L < 0 \) and the ball drops faster.

The final force considered in this model is the very small static buoyant force experienced by an object due to its being immersed in a fluid of non-zero density (anything other than a vacuum). The buoyant force is expressed in the fourth term of Equation 1, with \( \rho_b \) being the average density (mass per unit volume) of the ball. Now since Equation 1 is a vector differential equation it may be separated into three coupled scalar differential equations. However since this study focuses on motion in one plane only, the equation is separated into only two scalar equations - one for the horizontal (y) and the other for the vertical (z) directions:

\[
\frac{d v_y}{d t} = -\frac{C_d \rho \pi r^2 v_y}{2m} - \frac{\rho \pi r^3 \omega y}{2m} \tag{2a}
\]

\[
\frac{d v_z}{d t} = \left( \frac{\rho}{\rho_b} - 1 \right) g - \frac{C_d \rho \pi r^2 v_z}{2m} + \frac{\rho \pi r^3 \omega y}{2m}. \tag{2b}
\]

To obtain the trajectory of the ball and any other properties of interest, Equations (2a) and (2b) are integrated computationally.

### III. Computational Approach

Since Equations 2a and 2b are in fact coupled (they depend on each other), it is best to solve them as a computational initial value problem. The standard discretized kinematic equations suited for computational solution are

\[
v^{n+1} = v^n + a^n \Delta t \tag{3a}
\]

and

\[
x^{n+1} = x^n + v^n \Delta t + \frac{1}{2} a^n (\Delta t)^2. \tag{3b}
\]

Here the equations are applied to the y and z coordinates separately. The \( x^n \), \( v^n \) and \( a^n \) are the coordinates, velocities and accelerations (either horizontal or vertical components) of the ball at the \( n^{th} \) time step in the program. The time interval \( \Delta t \) is set equal to 0.01 seconds and is chosen so the results of the program validation are good. Two validation cases are used: the first is the case is the well-known situation where there are no effects from air present and the second is that of a baseball thrown without rotation.

The intent is to model different playing/practicing conditions vis-à-vis what adjustments a player would have to make in order to practice in one location and play in another where the average density of air is substantially different. The two different air densities used are shown in Appendix 1 (with other quantities to be referenced) and they approximate reasonable air densities at sea level and at high altitude (Colorado Springs, CO). The initial conditions for the position of
contacting the serve and ball velocity itself are entered into the program, as well as the net position, the net height, the clearance of the ball above the net and the final destination (range and height) of the ball. Varying initial conditions and net dimensions are utilized to model different playing environments for men and women. The algorithm then loops through initial speed and angle and determines the trajectory which most closely matches the net clearance and range simultaneously. The results include the ball’s trajectory as well as other quantities of interest. Subsequent to the determination of this trajectory, the program then changes the air density to a new value and calculates the trajectory of the ball with all other conditions (including hang time) the same. After the calculation of the second trajectory, certain quantities are shown to ensure that the conditions placed on the trajectory of the ball are met. For example, it is necessary to ensure that the ball actually did clear the net and land at its final destination in a manner reasonably consistent with the input.

The model in the study presented here assumes the air density is different, but could easily be extended so as to include the reasons for air density differences. Namely, the code could be restructured so that the air density is calculated from more accessible quantities like barometric pressure, altitude, temperature and humidity.

IV. Results
Appendix 1 contains values for the ball, air and gravity properties used in the simulations as well as the trajectory and player specifications used to model serves for both men and women. Appendix 2 contains important graphical data including ball paths for high altitude (pink curves) and for sea level (blue curves) for various topspins, initial (hit) ball speeds for a 50 foot range at both high altitude and sea level, average ball speeds for a 50 foot range at both places and hang times at both places. When ball speed is calculated as distance divided by time in a real game, it is the average speed, not the initial speed that is being measured. For the plots of the ball path, a 50-foot range trajectory is shown (accurate for all spins) and then the other curves of different color are calculations at the new air density with spin varying. In all cases, increasing spin causes increasing discrepancy between the ball destination and the destination for the 50-foot range curve. The nets are shown as yellow bars, and the area between red bars indicate likely ranges of spins, speeds and hang times for men and women separately.

V. Discussion of the Results

Explaining the results in the language of formulae, equations and physics laws is not going to be helpful to the volleyball audience. The following statements are the interpretations of the results that are relevant to the effect air density can have on the flight pattern of a serve.

1. The air density affects the trajectory of the serve.
2. The amount of change in the flight pattern of a serve differs when a man or woman serves.
3. For a given range, the serve trajectory patterns for different spins are not noticeably different.
4. The maximum changes in serve pattern are calculated between Colorado Springs, CO and sea level or from sea level to Colorado Springs.
5. The results of training in one altitude and performing in the other differ according to the altitude differences of the two locations.
6. For a given ball speed and flight time, varying the number of spins changes the height of the ball’s destination but not the distance it travels to any noticeable degree.
7. The topspin drop at the end of the topspin serve flight pattern is sharper at sea level than at high altitude. This effect is more pronounced when the number of spins increases.
8. Since larger topspin causes the ball to drop faster, the ball must be hit harder when the spin is larger to attain the same range as for a smaller spin at a given altitude.
9. With everything else being the same, the ball bust be hit harder to obtain the same range as altitude decreases.
10. The flight pattern could be adjusted to match other altitudes if the force creating the speed and the spin can be simultaneously adjusted.
11. In the men’s topspin serve, the difference in ball destination (the differences in positions of the ball at the same time) from sea level to Colorado Springs is about 23 inches longer horizontal and 6 inches higher vertical, based on a topspin of 10 rev/sec.
12. In the women’s topspin serve, the difference in ball destination from sea level to Colorado Springs is 23 inches longer horizontal and 5 inches higher vertical, based on a topspin of 8 rev/sec.
13. In the men’s topspin serve, the difference in ball destination from Colorado Springs to sea level is 25 inches shorter horizontal and 7 inches lower vertical.
14. In the women’s topspin serve, the difference in ball destination from Colorado Springs to sea level is about 22 inches shorter horizontal and 5 inches lower vertical.
15. Since the effect of changing air density is small compared to the entire ball path, coaches may estimate the changed in items (9) – (12) for any altitude difference by the following formula: (horizontal or vertical change in path for any altitude difference) = (horizontal or vertical change in path from sea level to Colorado springs) X (altitude difference in feet/5,280).

The results of this study advocates that the change in the topspin serve’s flight pattern from sea level to Colorado Springs or from Colorado Springs to sea level is significant. Since reading and quick reaction are the key components in pass accuracy, coaches should be concerned about training in one altitude and performing in another. Since the last few feet of the flight of a ball going to the arms is not directly observed, the position of the arms to pass in a predicted area as the training should be enforced. A sudden change of the flight pattern may cause passing problems resulting from training in one area and performing in another. The major recommendations of this work are as follows:

1. Women who train at sea level and compete at high altitude should have men serving to them at sea level to experience similar effects.
2. Men who train at sea level and compete at high altitude may train standing one or two feet inside the court in order to make the distance the ball travels shorter, requiring faster reaction and experiencing similar effects to high altitude.
3. Both men and women who train at high altitude and perform at sea level should serve slower at high altitude, with one or two rotations less.
4. It is advisable to train serve – passing a serve on several occasions at the sight of competition to get used to the atmospheric effects on the flight of the serve.
5. The results of this work are based on differing air density due to altitude. Factors such as barometric pressure, temperature and humidity also cause air density differences; the work presented here is easily extendable to include those effects explicitly.
6. There has been a FORTRAN program created that calculated the amount of trajectory change from altitude difference between any two locations. Get in touch with the authors for the solution. The recommendations made here are based on the findings of this research.
VI. References


3. CRC handbook, CRC Press.
Appendix 1

Ball, Air and Gravity Properties

Mass of ball: \( m = 0.27 \text{ kg} \) (ref. 2)
Radius of ball: \( r = 0.105 \text{ m} \) (ref. 2)
Drag coefficient: \( C_D = 0.5 \) (ref. 1)
High altitude air density: \( \rho = 1.03 \text{ kg/m}^3 \) (ref. 3)
Sea level air density: \( \rho = 1.20 \text{ kg/m}^3 \) (ref. 3)
Acceleration of gravity: \( g = 9.8 \text{ m/sec}^2 \) (well known)

Specifications for Men

Initial height of ball: \( z_0 = 10 \text{ ft.} \)
Final height of ball: \( z_f = 3 \text{ ft.} \)
Net position: \( y_{net} = 25 \text{ ft.} \)
Net height: \( z_{net} = 8 \text{ ft.} \)
Approximate clearance for bottom of ball at net: \( z_{clear} = 6 \text{ in.} \)
Ball range: \( y_f = 50 \text{ ft.} \)

Specifications for Women

Initial height of ball: \( z_0 = 9 \text{ ft.} \)
Final height of ball: \( z_f = 3 \text{ ft.} \)
Net position: \( y_{net} = 25 \text{ ft.} \)
Net height: \( z_{net} = 7.5 \text{ ft.} \)
Approximate clearance for bottom of ball at net: \( z_{clear} = 6 \text{ in.} \)
Ball range: \( y_f = 50 \text{ ft.} \)
Appendix 2

Graphical Data