1. Let $t \in [0, 1]$, $P(t) := \sum_{i=0}^n P_i B_i^n(t)$, and let $\tilde{P}(t) := \sum_{i=0}^n \tilde{P}_i B_i^n(t)$, where the control points $\tilde{P}_i$ are obtained from $P_i$ by moving the single point $P_j$ to $\tilde{P}_j$. Show that the difference $\tilde{P}(t) - P(t)$ is a vector pointing in the direction of $\tilde{P}_j - P_j$. Show that the amount of movement is positive for each $0 < t < 1$, and the largest at $t = j/n$. Hint: find an explicit expression for $\tilde{P}(t) - P(t)$. Illustrate your solution with graphs.

2. Degree Raising. $P(t) := \sum_{i=0}^n P_i B_i^n(t)$ can be rewritten as $P(t) := \sum_{i=0}^{n+1} \tilde{P}_i B_i^{n+1}(t)$, where

$$\tilde{P}_i = \frac{i}{n+1} P_{i-1} + \left(1 - \frac{i}{n+1}\right) P_i, \quad i = 0, \ldots, n+1$$

This can be obtained by multiplying $P(t)$ by $[t + (1-t)]$, expanding out and collecting terms.

Check correctness of the more general formula for degree raising from degree $n$ to degree $m$ :

$$\tilde{P}_i = \sum_{k=0}^n P_k \binom{n}{k} \binom{m-n}{i-k}, \quad i = 0, \ldots, m$$
using $m = n + 1$ and observing the pattern.

3. Construct an example of a Bézier curve that is convex, but whose control polygon is not convex. Hint: think about $y = x^4$ on $[-1, 1]$.

4. Give an example of a parametric curve $P(t)$ which is not convex but whose components $x(t)$ and $y(t)$ are convex. Illustrate with graphs.
5. Create an example of Bézier curve of degree 2. Subdivide the curve about the point \( t = 1/3 \) using the de Casteljeau algorithm, and graph the two parts of the curve on separate sets of axes.