

Chapter 20:

Exercises: 3, 7, 11, 22, 28, 34

EOC: 40, 43, 46, 58

E3:

A gasoline engine takes in  $1.80 \times 10^4 \text{ J}$  and delivers  $3800 \text{ J}$  of work per cycle. The heat is obtained by burning gasoline with a heat of combustion of  $4.60 \times 10^4 \text{ J/g}$ .

**Part A**

What is the thermal efficiency?

ANSWER: 
$$e = \frac{W}{Q}$$

**Part B**

How much heat is discarded in each cycle?

ANSWER: 
$$Q - W \quad \text{J}$$

**Part C**

What mass of fuel is burned in each cycle?

ANSWER: 
$$m = \frac{Q}{\Delta H} \quad \text{g}$$

**Part D**

If the engine goes through 50.0 cycles per second, what is its power output in kilowatts?

ANSWER: 
$$\frac{Wn}{1000} \quad \text{kW}$$

**Part E**

If the engine goes through 50.0 cycles per second, what is its power output in horsepower?

ANSWER: 
$$\frac{1.342Wn}{1000} \quad \text{hp}$$

E7:

Part A

What compression ratio  $r$  must an Otto cycle have to achieve an ideal efficiency of 66.2 % if  $\gamma = 1.45$ ?

**ANSWER:**  $r = (1 - \eta)^{\frac{1}{1-\gamma}}$

E11:

A window air-conditioner unit absorbs  $9.80 \times 10^4 \text{ J}$  of heat per minute from the room being cooled and in the same time period deposits  $1.50 \times 10^5 \text{ J}$  of heat into the outside air.

**Part A**

What is the power consumption of the unit?

**ANSWER:**  $\frac{Q_2 - Q_1}{60} \text{ W}$

**Part B**

What is the energy efficiency rating of the unit?

**ANSWER:**  $EER = \frac{Q_1}{Q_2 - Q_1} \cdot 3.413$

E22:

**Description:** A Carnot heat engine uses a hot reservoir consisting of a large amount of boiling water and a cold reservoir consisting of a large tub of ice and water. In five minutes of operation of the engine, the heat rejected by the engine melts 0.0400 kg of...

A Carnot heat engine uses a hot reservoir consisting of a large amount of boiling water and a cold reservoir consisting of a large tub of ice and water. In five minutes of operation of the engine, the heat rejected by the engine melts 0.0400 kg of ice.

**Part A**

During this time, how much work  $W$  is performed by the engine?

**ANSWER:**  $W = 4890 \text{ J}$

E28:

**Description:** You make tea with 0.250 kg of T water and let it cool to room temperature (20.0 degree(s) C) before drinking it. (a) Calculate the entropy change of the water while it cools. (b) The cooling process is essentially isothermal for the air in your...

You make tea with 0.250 kg of 86.0 °C water and let it cool to room temperature (20.0 °C) before drinking it.

**Part A**

Calculate the entropy change of the water while it cools.

**ANSWER:**

$$\Delta S = 0.25 \cdot 4190 \ln\left(\frac{293.2}{T}\right) \text{ J/K}$$

**Part B**

The cooling process is essentially isothermal for the air in your kitchen. Calculate the change in entropy of the air while the tea cools, assuming that all the heat lost by the water goes into the air.

**ANSWER:**

$$\Delta S = \frac{0.25 \cdot 4190 (T - 293.2)}{293.2} \text{ J/K}$$

**Part C**

What is the total entropy change of the system tea + air?

Express your answer using two significant figures.

**ANSWER:**

$$\Delta S = \frac{0.25 \cdot 4190 (T - 293.2)}{293.2} + 0.25 \cdot 4190 \ln\left(\frac{293.2}{T}\right) \text{ J/K}$$

E34:

**Description:** A box is separated by a partition into two parts of equal volume. The left side of the box contains 500 molecules of nitrogen gas; the right side contains 100 molecules of oxygen gas. The two gases are at the same temperature. The partition is...

A box is separated by a partition into two parts of equal volume. The left side of the box contains 500 molecules of nitrogen gas; the right side contains 100 molecules of oxygen gas. The two gases are at the same temperature. The partition is punctured, and equilibrium is eventually attained. Assume that the volume of the box is large enough for each gas to undergo a free expansion and not change temperature.

**Part A**

On average, how many molecules of oxygen will there be in either half of the box?  
Express your answer using two significant figures.

**ANSWER:**    50  
                  molecules

**Part B**

On average, how many molecules of nitrogen will there be in either half of the box?  
Express your answer using two significant figures.

**ANSWER:**    250  
                  molecules

**Part C**

What is the change in entropy of the system when the partition is punctured?

**ANSWER:**     $\Delta S = 5.74 \cdot 10^{-21} \text{ J/K}$

**Part D**

What is the probability that the molecules will be found in the same distribution as they were before the partition was punctured, that is, 500 nitrogen molecules in the left half and 100 oxygen molecules in the right half? Give the *order of magnitude*.

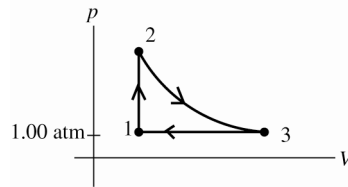
Express your answer as the order of magnitude. If your answer is  $3 \times 10^{-73}$ , then the order of magnitude is  $-73$ , so you would enter  $-73$

**ANSWER:**     $-181$

**20.40. IDENTIFY:** Use the ideal gas law to calculate  $p$  and  $V$  for each state. Use the first law and specific expressions for  $Q$ ,  $W$ , and  $\Delta U$  for each process. Use Eq.(20.4) to calculate  $e$ .  $Q_H$  is the net heat flow into the gas.

**SET UP:**  $\gamma = 1.40$

$C_V = R/(\gamma - 1) = 20.79 \text{ J/mol} \cdot \text{K}$ ;  $C_p = C_V + R = 29.10 \text{ J/mol} \cdot \text{K}$ . The cycle is sketched in Figure 20.40.



$$T_1 = 300 \text{ K}$$

$$T_2 = 600 \text{ K}$$

$$T_3 = 492 \text{ K}$$

**Figure 20.40**

**EXECUTE: (a) point 1**

$$p_1 = 1.00 \text{ atm} = 1.013 \times 10^5 \text{ Pa (given); } pV = nRT;$$

$$V_1 = \frac{nRT_1}{p_1} = \frac{(0.350 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(300 \text{ K})}{1.013 \times 10^5 \text{ Pa}} = 8.62 \times 10^{-3} \text{ m}^3$$

**point 2**

process  $1 \rightarrow 2$  at constant volume so  $V_2 = V_1 = 8.62 \times 10^{-3} \text{ m}^3$

$$pV = nRT \text{ and } n, R, V \text{ constant implies } p_1/T_1 = p_2/T_2$$

$$p_2 = p_1(T_2/T_1) = (1.00 \text{ atm})(600 \text{ K}/300 \text{ K}) = 2.00 \text{ atm} = 2.03 \times 10^5 \text{ Pa}$$

**point 3**

Consider the process  $3 \rightarrow 1$ , since it is simpler than  $2 \rightarrow 3$ .

Process  $3 \rightarrow 1$  is at constant pressure so  $p_3 = p_1 = 1.00 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$

$$pV = nRT \text{ and } n, R, p \text{ constant implies } V_1/T_1 = V_3/T_3$$

$$V_3 = V_1(T_3/T_1) = (8.62 \times 10^{-3} \text{ m}^3)(492 \text{ K}/300 \text{ K}) = 14.1 \times 10^{-3} \text{ m}^3$$

**(b) process  $1 \rightarrow 2$**

constant volume ( $\Delta V = 0$ )

$$Q = nC_V\Delta T = (0.350 \text{ mol})(20.79 \text{ J/mol} \cdot \text{K})(600 \text{ K} - 300 \text{ K}) = 2180 \text{ J}$$

$$\Delta V = 0 \text{ and } W = 0. \text{ Then } \Delta U = Q - W = 2180 \text{ J}$$

**process  $2 \rightarrow 3$**

Adiabatic means  $Q = 0$ .

$$\Delta U = nC_V\Delta T \text{ (any process), so}$$

$$\Delta U = (0.350 \text{ mol})(20.79 \text{ J/mol} \cdot \text{K})(492 \text{ K} - 600 \text{ K}) = -780 \text{ J}$$

Then  $\Delta U = Q - W$  gives  $W = Q - \Delta U = +780 \text{ J}$ . (It is correct for  $W$  to be positive since  $\Delta V$  is positive.)

**process  $3 \rightarrow 1$**

For constant pressure

$$W = p\Delta V = (1.013 \times 10^5 \text{ Pa})(8.62 \times 10^{-3} \text{ m}^3 - 14.1 \times 10^{-3} \text{ m}^3) = -560 \text{ J}$$

or  $W = nR\Delta T = (0.350 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(300 \text{ K} - 492 \text{ K}) = -560 \text{ J}$ , which checks. (It is correct for  $W$  to be negative, since  $\Delta V$  is negative for this process.)

$$Q = nC_p\Delta T = (0.350 \text{ mol})(29.10 \text{ J/mol} \cdot \text{K})(300 \text{ K} - 492 \text{ K}) = -1960 \text{ J}$$

$$\Delta U = Q - W = -1960 \text{ J} - (-560 \text{ J}) = -1400 \text{ J}$$

or  $\Delta U = nC_V\Delta T = (0.350 \text{ mol})(20.79 \text{ J/mol} \cdot \text{K})(300 \text{ K} - 492 \text{ K}) = -1400 \text{ J}$ , which checks

$$\text{(c) } W_{\text{net}} = W_{1 \rightarrow 2} + W_{2 \rightarrow 3} + W_{3 \rightarrow 1} = 0 + 780 \text{ J} - 560 \text{ J} = +220 \text{ J}$$

$$\text{(d) } Q_{\text{net}} = Q_{1 \rightarrow 2} + Q_{2 \rightarrow 3} + Q_{3 \rightarrow 1} = 2180 \text{ J} + 0 - 1960 \text{ J} = +220 \text{ J}$$

$$\text{(e) } e = \frac{\text{work output}}{\text{heat energy input}} = \frac{W}{Q_H} = \frac{220 \text{ J}}{2180 \text{ J}} = 0.101 = 10.1\%$$

$$e(\text{Carnot}) = 1 - T_C/T_H = 1 - 300 \text{ K}/600 \text{ K} = 0.500.$$

**EVALUATE:** For a cycle  $\Delta U = 0$ , so by  $\Delta U = Q - W$  it must be that  $Q_{\text{net}} = W_{\text{net}}$  for a cycle. We can also check that  $\Delta U_{\text{net}} = 0$ :

$$\Delta U_{\text{net}} = \Delta U_{1 \rightarrow 2} + \Delta U_{2 \rightarrow 3} + \Delta U_{3 \rightarrow 1} = 2180 \text{ J} - 1050 \text{ J} - 1130 \text{ J} = 0$$

$e < e(\text{Carnot})$ , as it must.

**20.43. IDENTIFY:**  $T_b = T_c$  and is equal to the maximum temperature. Use the ideal gas law to calculate  $T_a$ . Apply the appropriate expression to calculate  $Q$  for each process.  $e = \frac{W}{Q_H}$ .  $\Delta U = 0$  for a complete cycle and for an isothermal process of an ideal gas.

**SET UP:** For helium,  $C_V = 3R/2$  and  $C_p = 5R/2$ . The maximum efficiency is for a Carnot cycle, and  $e_{\text{Carnot}} = 1 - T_C/T_H$ .

**EXECUTE:** (a)  $Q_{\text{in}} = Q_{ab} + Q_{bc}$ .  $Q_{\text{out}} = Q_{ca}$ .  $T_{\text{max}} = T_b = T_c = 327^\circ\text{C} = 600\text{ K}$ .

$$\frac{p_a V_a}{T_a} = \frac{p_b V_b}{T_b} \rightarrow T_a = \frac{p_a T_b}{p_b} = \frac{1}{3}(600\text{ K}) = 200\text{ K}. \quad p_b V_b = nRT_b \rightarrow V_b = \frac{nRT_b}{p_b} = \frac{(2\text{ moles})(8.31\text{ J/mol}\cdot\text{K})(600\text{ K})}{3.0 \times 10^5\text{ Pa}} = 0.0332\text{ m}^3.$$

$$\frac{p_b V_b}{T_b} = \frac{p_c V_c}{T_c} \rightarrow V_c = V_b \frac{p_b}{p_c} = (0.0332\text{ m}^3) \left( \frac{3}{1} \right) = 0.0997\text{ m}^3 = V_a.$$

$$Q_{ab} = nC_V \Delta T_{ab} = (2\text{ mol}) \left( \frac{3}{2} \right) (8.31\text{ J/mol}\cdot\text{K})(400\text{ K}) = 9.97 \times 10^3\text{ J}$$

$$Q_{bc} = W_{bc} = \int_b^c p dV = \int_b^c \frac{nRT_b}{V} dV = nRT_b \ln \frac{V_c}{V_b} = nRT_b \ln 3.$$

$$Q_{bc} = (2.00\text{ mol})(8.31\text{ J/mol}\cdot\text{K})(600\text{ K}) \ln 3 = 1.10 \times 10^4\text{ J}. \quad Q_{\text{in}} = Q_{ab} + Q_{bc} = 2.10 \times 10^4\text{ J}.$$

$$Q_{\text{out}} = Q_{ca} = nC_p \Delta T_{ca} = (2.00\text{ mol}) \left( \frac{5}{2} \right) (8.31\text{ J/mol}\cdot\text{K})(400\text{ K}) = 1.66 \times 10^4\text{ J}.$$

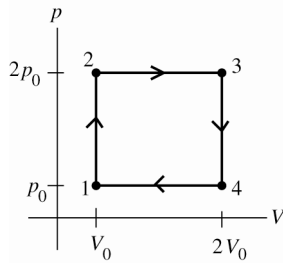
(b)  $Q = \Delta U + W = 0 + W \rightarrow W = Q_{\text{in}} - Q_{\text{out}} = 2.10 \times 10^4\text{ J} - 1.66 \times 10^4\text{ J} = 4.4 \times 10^3\text{ J}.$

$$e = W/Q_{\text{in}} = \frac{4.4 \times 10^3\text{ J}}{2.10 \times 10^4\text{ J}} = 0.21 = 21\%.$$

(c)  $e_{\text{max}} = e_{\text{Carnot}} = 1 - \frac{T_C}{T_H} = 1 - \frac{200\text{ K}}{600\text{ K}} = 0.67 = 67\%$

**EVALUATE:** The thermal efficiency of this cycle is about one-third of the efficiency of a Carnot cycle that operates between the same two temperatures.

- 20.46. IDENTIFY:** Use Eq.(20.4) to calculate  $e$ .  
**SET UP:** The cycle is sketched in Figure 20.46.



**Figure 20.46**

$$C_V = 5R/2$$

for an ideal gas  $C_p = C_V + R = 7R/2$

**SET UP:** Calculate  $Q$  and  $W$  for each process.

process 1  $\rightarrow$  2

$$\Delta V = 0 \text{ implies } W = 0 \text{ \& } \Delta V = 0 \text{ implies } Q = nC_V \Delta T = nC_V (T_2 - T_1)$$

But  $pV = nRT$  and  $V$  constant says  $p_1 V = nRT_1$  and  $p_2 V = nRT_2$ .

Thus  $(p_2 - p_1)V = nR(T_2 - T_1)$ ;  $V\Delta p = nR\Delta T$  (true when  $V$  is constant).

Then  $Q = nC_V \Delta T = nC_V (V\Delta p/nR) = (C_V/R)V\Delta p = (C_V/R)V_0(2p_0 - p_0) = (C_V/R)p_0V_0$ .  $Q > 0$ ; heat is absorbed by the gas.)

process 2  $\rightarrow$  3

$$\Delta p = 0 \text{ so } W = p\Delta V = p(V_3 - V_2) = 2p_0(2V_0 - V_0) = 2p_0V_0 \text{ (} W \text{ is positive since } V \text{ increases.)}$$

$$\Delta p = 0 \text{ implies } Q = nC_p \Delta T = nC_p (T_2 - T_1)$$

But  $pV = nRT$  and  $p$  constant says  $pV_1 = nRT_1$  and  $pV_2 = nRT_2$ .

Thus  $p(V_2 - V_1) = nR(T_2 - T_1)$ ;  $p\Delta V = nR\Delta T$  (true when  $p$  is constant).

Then  $Q = nC_p \Delta T = nC_p (p\Delta V/nR) = (C_p/R)p\Delta V = (C_p/R)2p_0(2V_0 - V_0) = (C_p/R)2p_0V_0$ . ( $Q > 0$ ; heat is absorbed by the gas.)

process 3  $\rightarrow$  4

$$\Delta V = 0 \text{ implies } W = 0$$

$\Delta V = 0$  so

$$Q = nC_V \Delta T = nC_V (V\Delta p/nR) = (C_V/R)(2V_0)(p_0 - 2p_0) = -2(C_V/R)p_0V_0$$

( $Q < 0$  so heat is rejected by the gas.)

process 4  $\rightarrow$  1

$$\Delta p = 0 \text{ so } W = p\Delta V = p(V_1 - V_4) = p_0(V_0 - 2V_0) = -p_0V_0 \text{ (} W \text{ is negative since } V \text{ decreases)}$$

$$\Delta p = 0 \text{ so } Q = nC_p \Delta T = nC_p (p\Delta V/nR) = (C_p/R)p\Delta V = (C_p/R)p_0(V_0 - 2V_0) = -(C_p/R)p_0V_0 \text{ (} Q < 0 \text{ so heat is rejected by the gas.)}$$

total work performed by the gas during the cycle:

$$W_{\text{tot}} = W_{1 \rightarrow 2} + W_{2 \rightarrow 3} + W_{3 \rightarrow 4} + W_{4 \rightarrow 1} = 0 + 2p_0V_0 + 0 - p_0V_0 = p_0V_0$$

(Note that  $W_{\text{tot}}$  equals the area enclosed by the cycle in the  $pV$ -diagram.)

total heat absorbed by the gas during the cycle ( $Q_H$ ): Heat is absorbed in processes 1  $\rightarrow$  2 and 2  $\rightarrow$  3.

$$Q_H = Q_{1 \rightarrow 2} + Q_{2 \rightarrow 3} = \frac{C_V}{R} p_0V_0 + 2 \frac{C_p}{R} p_0V_0 = \left( \frac{C_V + 2C_p}{R} \right) p_0V_0$$

$$\text{But } C_p = C_V + R \text{ so } Q_H = \frac{C_V + 2(C_V + R)}{R} p_0V_0 = \left( \frac{3C_V + 2R}{R} \right) p_0V_0.$$

total heat rejected by the gas during the cycle ( $Q_C$ ): Heat is rejected in processes 3  $\rightarrow$  4 and 4  $\rightarrow$  1.

$$Q_C = Q_{3 \rightarrow 4} + Q_{4 \rightarrow 1} = -2 \frac{C_V}{R} p_0V_0 - \frac{C_p}{R} p_0V_0 = - \left( \frac{2C_V + C_p}{R} \right) p_0V_0$$

$$\text{But } C_p = C_V + R \text{ so } Q_C = - \frac{2C_V + (C_V + R)}{R} p_0V_0 = - \left( \frac{3C_V + R}{R} \right) p_0V_0.$$

$$\text{efficiency } \rightarrow e = \frac{W}{Q_H} = \frac{p_0V_0}{\left( \frac{3C_V + 2R}{R} \right) p_0V_0} = \frac{R}{3C_V + 2R} = \frac{R}{3(5R/2) + 2R} = \frac{2}{19}. e = 0.105 = 10.5\%$$

**EVALUATE:** As a check on the calculations note that  $Q_C + Q_H = - \left( \frac{3C_V + R}{R} \right) p_0V_0 + \left( \frac{3C_V + 2R}{R} \right) p_0V_0 = p_0V_0 = W$ , as it should.

**20.58. IDENTIFY and SET UP:** First use the methods of Chapter 17 to calculate the final temperature  $T$  of the system.

**EXECUTE:** 0.600 kg of water (cools from  $45.0^\circ\text{C}$  to  $T$ )

$$Q = mc\Delta T = (0.600 \text{ kg})(4190 \text{ J/kg} \cdot \text{K})(T - 45.0^\circ\text{C}) = (2514 \text{ J/K})T - 1.1313 \times 10^5 \text{ J}$$

0.0500 kg of ice (warms to  $0^\circ\text{C}$ , melts, and water warms from  $0^\circ\text{C}$  to  $T$ )

$$Q = mc_{\text{ice}}(0^\circ\text{C} - (-15.0^\circ\text{C})) + mL_f + mc_{\text{water}}(T - 0^\circ\text{C})$$

$$Q = 0.0500 \text{ kg} \left[ (2100 \text{ J/kg} \cdot \text{K})(15.0^\circ\text{C}) + 334 \times 10^3 \text{ J/kg} + (4190 \text{ J/kg} \cdot \text{K})(T - 0^\circ\text{C}) \right]$$

$$Q = 1575 \text{ J} + 1.67 \times 10^4 \text{ J} + (209.5 \text{ J/K})T = 1.828 \times 10^4 \text{ J} + (209.5 \text{ J/K})T$$

$$Q_{\text{system}} = 0 \text{ gives } (2514 \text{ J/K})T - 1.1313 \times 10^5 \text{ J} + 1.828 \times 10^4 \text{ J} + (209.5 \text{ J/K})T = 0$$

$$(2.724 \times 10^3 \text{ J/K})T = 9.485 \times 10^4 \text{ J}$$

$$T = (9.485 \times 10^4 \text{ J}) / (2.724 \times 10^3 \text{ J/K}) = 34.83^\circ\text{C} = 308 \text{ K}$$

**EVALUATE:** The final temperature must lie between  $-15.0^\circ\text{C}$  and  $45.0^\circ\text{C}$ . A final temperature of  $34.8^\circ\text{C}$  is consistent with only liquid water being present at equilibrium.

**IDENTIFY and SET UP:** Now we can calculate the entropy changes. Use  $\Delta S = Q/T$  for phase changes and the method of Example 20.6 to calculate  $\Delta S$  for temperature changes.

**EXECUTE:** ice: The process takes ice at  $-15^\circ\text{C}$  and produces water at  $34.8^\circ\text{C}$ . Calculate  $\Delta S$  for a reversible process between these two states, in which heat is added very slowly.  $\Delta S$  is path independent, so  $\Delta S$  for a reversible process is the same as  $\Delta S$  for the actual (irreversible) process as long as the initial and final states are the same.

$$\Delta S = \int_1^2 dQ/T, \text{ where } T \text{ must be in kelvins}$$

For a temperature change  $dQ = mc dT$  so  $\Delta S = \int_{T_1}^{T_2} (mc/T) dT = mc \ln(T_2/T_1)$ .

For a phase change, since it occurs at constant  $T$ ,

$$\Delta S = \int_1^2 dQ/T = Q/T = \pm mL/T.$$

Therefore  $\Delta S_{\text{ice}} = mc_{\text{ice}} \ln(273 \text{ K}/258 \text{ K}) + mL_f/273 \text{ K} + mc_{\text{water}} \ln(308 \text{ K}/273 \text{ K})$

$$\Delta S_{\text{ice}} = (0.0500 \text{ kg})[(2100 \text{ J/kg} \cdot \text{K}) \ln(273 \text{ K}/258 \text{ K}) + (334 \times 10^3 \text{ J/kg})/273 \text{ K} + (4190 \text{ J/kg} \cdot \text{K}) \ln(308 \text{ K}/273 \text{ K})]$$

$$\Delta S_{\text{ice}} = 5.93 \text{ J/K} + 61.17 \text{ J/K} + 25.27 \text{ J/K} = 92.4 \text{ J/K}$$

water:  $\Delta S_{\text{water}} = mc \ln(T_2/T_1) = (0.600 \text{ kg})(4190 \text{ J/kg} \cdot \text{K}) \ln(308 \text{ K}/318 \text{ K}) = -80.3 \text{ J/K}$

For the system,  $\Delta S = \Delta S_{\text{ice}} + \Delta S_{\text{water}} = 92.4 \text{ J/K} - 80.3 \text{ J/K} = +12 \text{ J/K}$

**EVALUATE:** Our calculation gives  $\Delta S > 0$ , as it must for an irreversible process of an isolated system.