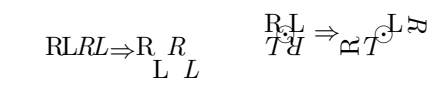
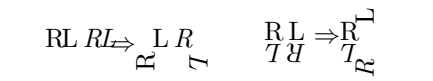
There are three important forms of sameness or symmetry which keep one off one's partner's feet, and move a couple where they are supposed to go in dance: same footwork, opposite footwork, and same footwork out of phase. The nature of motion (or stasis) depends on where one is in relation to one's partner (facing one's partner or facing same direction) and the direction of the footwork.

Same footwork (by same footwork I mean not just the same foot, but the same position relative to the body). If one is next to one's partner, the same footwork is often necessary so one does not interfere with one's partner (think of a can-can line, where the feet must kick off in the same direction; I think of such dance as tesselation in motion). With the same footwork when one is adjacent to one's partner, one can move anywhere on the floor with one's partner, but unless different size steps are allowed, if they stay next to each other, they cannot change the direction they are facing (but see below). If one is facing one'e partner and using the same footwork, one cannot change the location of the couple, their positions are always symmetric to the center point between them. Think of a buzz turn. This is called symmetry with respect to the origin (the point between them) and is the same concept as odd functions. The first schematic below represents same footwork when one is adjacent to one's partner, the second when one is facing one's partner (with the center designated with a circle with a dot).



Opposite footwork (By opposite footwork I mean not just the opposite (right corresponding to left) foot, but the mirror image position relative to the body). With opposite footwork when one is next to one's partner, one can go anywhere along the line between the two partners; the partners will always be mirror image across that line. Think of the open conversation in fox trot. (This is the essence of an even function). Opposite footwork when one is facing ones partner also provides mirror image across the line between the partners. See the scematics below. The mirror image lines are vertical and horizontal, respectively.



Same footwork out of phase. Several dances such as the waltz (see basic box step schematic below) work by having both partners do the same footwork, but out of phase. This may entail the couple moving back and fourth as in the waltz box step or cha cha, or progressing along a line as with the waltz turn, hambo, or polka (the polka is opposite footwork, but the turns preclude it from being mirror image; it is the same step out of phase including the turns).

Questions: How can a couple be same footwork and opposite footwork at the same time? Same footwork and same footwork out of phase? opposite footwork and same footwork out of phase? [Because your must have the same footwork as your partner, and the mirror image of that with your other foot, each partner must be a mirror image of him/herself down the middle, and all motion for each dancer must be along a line through the middle of that dancer. If they are facing each other, because they have the same footwork, they cannot change their position on the floor as a couple, nor can they move around a circle because each remains symmetric with respect to the line through his/her middle. This does not mean they cannot move at all, they can do jumping jacks facing each other. If they are next to each other, they can move forward or back along the line between them (which is parallel to the lines through each of them), but they must move by jumping (both feet move together symmetrically). Of course, if one is behind the other, they can both do the bunny hop. Opposite footwork is the same as same footwork out of phase if they are just alternating steps which are mirror images of each other (e.g., walking forward or rocking side to side). Same footwork is the same footwork in phase, or off by a cycle if the footwork is repeated (this will include buzz turns).]

Where can the partners as individuals move on the floor with same footwork facing, same footwork adjacent, opposite footwork facing, opposite footwork adjacent, same footwork out of

phase? (the above discussion was about the movement of a point in some sense centered between them). [Same footwork facing: (assuming they are not holding hands) they can spiral out (while still facing each other) and thereby reach any point on the floor as an individual. Same footwork adjacent: we already mentioned above that they can move to any point as a couple. Opposite footwork facing: they could back up and and move to either side while still facing, but could not physically pass through each other, so they are restricted to their own side of the line between them. Opposite footwork adjacent: there is again the problem of physically passing between them, so they are restricted to their side of the line between them. Same footwork out of phase: Since same footwork in phase is a special case of this, they could move to any point on the floor as individuals (for example the buzz step which repeats itself every step is in phase or out of phase by 1, 2, 3, etc. steps; the angle of a spiral would have to be set and held constant for the steps to be the same.)

Transitions between symmetries.

Dances of course include several of the above symmetries as well as steps which do not manifest any of the above symmetries. There may need to be transitions from one mode to another. We now consider some possible transitions.

Same footwork, facing to adjacent: This will require an interruption in the same footwork, since one partner will need to turn more than the other to make them adjacent (or one will turn right while the other turns left which is not consistent with same footwork); they can each turn 90 degrees and be adjacent, but that will leave them facing opposite directions.

Opposite footwork facing to adjacent: This will be easy, while facing they just both turn 90 degrees in opposite directions, which is consistent with opposite footwork. This can be used as a means to change from same footwork facing to same footwork adjacent: while facing change to opposite footwork, turn to adjacent, then change back to same footwork.

Facing or Adjacent, same to opposite footwork: These transitions obviously just entail changing footwork.

Out of phase: As mentioned above, when partners are facing (which is where most out of phase starts, one can just transition between same footwork, opposite footwork, or out of phase. Also note that we defined same footwork and opposite footwork for facing or adjacent, opposite footwork may change from facing to adjacent, which may also be interpreted as out of phase. The cha cha is an example where a basic out of phase dance has a variation which can be interpreted as out of phase or mirror image (the side to side step).